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# Perturbative Regge asymptotics in the case of non-vacuum exchange <sup>1</sup>

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## **Abstract**

Some results on the perturbative Regge asymptotics are reviewed. The concepts of the reggeon interaction approach and the double logarithmic approximation are outlined.

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# 1 Results on non-vacuum exchange

The leading logarithmic ( $\ln s$ ) and the double-logarithmic approximation schemes provide results about the perturbative Regge asymptotics. These results apply to semi-hard processes where besides of the large energy  $s$  there is a second large scale  $Q^2$  controlling the coupling,  $s \gg Q^2 \gg \Lambda_{QCD}^2$ . The predictions for the small  $x$  behaviour of structure functions and on features of the hadronic final state in deep-inelastic scattering at small  $x$  are in qualitative agreement with experiment and usually attention is paid to the features deviating from the GLAP evolution. On the other hand the  $Q^2$  dependence of the small  $x$  asymptotic results is determined by the asymptotics of the anomalous dimensions for the continued moment number  $j$  (angular momentum) approaching some integer value. The relation of the perturbative Regge results to anomalous dimensions is well known [1, 2]. These asymptotic anomalous dimensions involving the contributions from all orders are useful for checking complete higher order calculations and for improving the kernels in the GLAP evolution [3, 4].

The perturbative Regge asymptotics for the vacuum exchange channel, the BFKL pomeron [5], has now its place in semi-hard phenomenology. It is natural to look for situations where the asymptotics of other exchange channels shows up.

The case of flavour non-singlet exchange dominated by fermion and antifermion with the same chirality in the  $t$  channel has been studied for many years. The first methods of double-logarithmic approximation to the perturbative Regge asymptotics have been worked out in QED [6] and partially extended to QCD [7]. The method of separation of the softest particle [2] provided an essential simplification and allowed to treat both signature channels.

Alternatively the two-fermion exchange can be treated in close analogy to the two-gluon exchange of the BFKL pomeron [8]. In this way all contributions with  $\ln s$  from the longitudinal part of each loop integral are summed up. There may be further single  $\ln s$  contributions. In this approach the negative signature case has not been treated yet, because it is related to a three reggeon exchange.

The positive signature part of this channel contributes to the flavour non-singlet  $F_2(x, Q^2)$  structure function [2]. The application of the negative signature part to the flavour non-singlet part of the spin structure function  $g_1(x, Q^2)$  has been considered first in [9].

The flavour singlet part of  $g_1(x, Q^2)$  corresponds to odd parity exchange. Besides of the quarks also two gluons can be exchanged. However at least one of the two gluons is of a different type (helicity state) compared to the one generating the asymptotics in the vacuum (BFKL) channel [10]. The leading double log asymptotics has been worked out recently [11]. Both quark-antiquark and two-gluon intermediate reggeon states give rise to a singularity near  $j = 0$  and the mixing of these channels has to be considered.

The positive signature part of the odd parity channel can be related to the spin counterpart of the structure function  $F_3(x, Q^2)$ , which is called  $g_5$  [12] or  $a_1$  [13].

Odd parity can in principle be transferred by more than two leading (BFKL type) gluons. The corresponding coupling structure (impact factor) is not known and is eventually related to higher twist. This would give rise to a singularity near  $j = 1$  beyond the leading logarithmic approximation.

From studying the multi-Regge effective action [17] it is evident that there are two types of fermion exchanges characterized by chirality or helicity and resulting in the same asymptotics. The resulting two kernels of reggeized two-fermion interactions have been treated in analogy to the BFKL case [8]. The case with equal chirality of the two fermions has been mentioned above. The other case of opposite chirality (parallel helicity) of the exchanged quark and antiquark shows some unusual features [14]. The leading Regge asymptotics (near  $j = 0$ ) has no double logarithmic contributions. Double logs contribute at the subleading ( $j = -1$ ) level and are connected to the (one - loop) GLAP evolution. These features are expected to show up in the structure function  $h_1(x, Q^2)$  which appears in the Drell-Yan process and measures the transversity distribution [15].

## 2 The reggeon interaction approach

The multi-Regge effective action is a convenient formulation of the leading  $\ln s$  approximation, i.e. for summing contributions with the leading logarithm arising from the longitudinal momentum integration. It has been proposed [16] as a starting point for investigating unitarity corrections (multi-reggeon exchange with multi-Regge kinematics in all  $s$ -channels) and genuine next-to-leading logarithmic corrections (quasi-multi-Regge kinematics). This action can be derived from the QCD action by separating the gluon and the quark fields into modes according to the multi-Regge kinematics. The "heavy" modes, which do not correspond to exchanged or scattered particles, are eliminated. The exchanged modes correspond to momenta  $k$  with

$$k_+ k_- \ll |\kappa^2|, \quad (1)$$

where  $k_{\pm}$  are the light cone components (essentially in the direction of the incoming particles) and  $\kappa$  are the transverse components. In the light-cone axial gauge the transverse components of the vector potential  $A_{\sigma}, \sigma = 1, 2$  describes the physical degrees of freedom of the gluon. There are two types of exchange fields expressed in terms of the modes (1) of  $A_{\sigma}$ ,

$$\begin{aligned} \mathcal{A}_+ &= \partial_-^{-1} \partial_{\sigma} A_{\sigma} \quad (\text{G}), \\ \mathcal{A}' &= i\epsilon_{\rho\sigma} \partial_{\sigma} A_{\rho} \quad (\text{G}^{\perp}). \end{aligned} \quad (2)$$

The first exchange field gives rise to the leading asymptotics ( $\mathcal{O}(s^{1+\omega}), \omega = \mathcal{O}(g^2)$ ), whereas the exchange of each  $\mathcal{A}'$  leads to a suppression by one factor of  $s^{-1}$ . There are also two types of fermion exchanges characterized by the two chirality projections. Each of them leads to a suppression by a factor of  $s^{-1/2}$  compared to the leading asymptotics. The two gluon and the two fermion exchanges can be related by supersymmetry.

The terms in the effective action describing the non-leading  $\mathcal{A}'$  exchange are not completely known yet. The effective production vertices with the other three exchanges give rise to the following two-reggeon interaction kernels [8],

$$\begin{aligned} K_{GG} &= |\kappa_1 - \kappa'_1|^2 \left( \frac{\kappa_1 \kappa_2^*}{\kappa'_1 \kappa'^*_2} + \text{c.c.} \right), \\ K_{F\overline{F}} &= |\kappa_1 - \kappa'_1|^{-2} \left( \left| \frac{\kappa_1}{\kappa'_1} \right|^{\omega} + \left| \frac{\kappa_1}{\kappa'_1} \right|^{-\omega} \frac{\kappa_1^* \kappa_2}{\kappa'^*_1 \kappa'_2} \right), \end{aligned}$$

$$K_{FF} = |\kappa_1 - \kappa'_1|^{-2} \left( \frac{\kappa_2^*}{\kappa_2'^*} + \frac{\kappa_1^*}{\kappa_1'^*} \right). \quad (3)$$

The kernel  $K_{GF}$  is not needed for our discussion here. The first expression in (3) is the well known BFKL kernel [5] in the complex notation for the transverse momentum vectors. The momentum transfer is  $\kappa_1 + \kappa_2 = \kappa'_1 + \kappa'_2$ . The second kernel  $K_{F\overline{F}}$  corresponds to the equal chirality exchange. The  $\omega$ -dependent factors appear due to the double logarithmic behaviour at  $\kappa \ll \kappa'$  and at  $\kappa \gg \kappa'$ . The third kernel describes opposite chirality exchange and does not lead to a logarithmic transverse momentum integral.

Taking the reggeization into account the kernels are modified and become infrared finite operators,

$$H_{ij} = K_{ij} - \alpha_i(\kappa_1)\delta(\kappa_1 - \kappa'_1) - \alpha_j(\kappa_2)\delta(\kappa_2 - \kappa'_2). \quad (4)$$

$\alpha_i(\kappa)$  is the one-loop trajectory function of the gluon ( $i = G$ ) or the quark ( $i = F, \overline{F}$ ). The equations and the resulting  $2 \rightarrow 2$  reggeon Green functions are analogous to the BFKL case. We restrict ourselves to the case of vanishing momentum transfer ( $\kappa_1 = -\kappa_2 = \kappa$ ).

$$\frac{1}{2\pi^2} \sum_{r=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \frac{f^{(ij)}(\omega, \kappa, \overline{\kappa})}{\omega - \frac{g^2 C^{(ij)}}{8\pi^2} \Omega^{(ij)}(n, \nu, \omega)} = \quad (5)$$

$\Omega^{(ij)}(n, \nu, \omega)$  are the eigenvalues of the kernel  $H_{ij}$  and  $f_{n,\nu}^{(ij)}$  are the corresponding eigenfunctions [8],

$$f_{n,\nu}^{(ij)}(\kappa) = (d_i(\kappa)d_j(\kappa))^{-1/2} |\kappa|^{-1/2+i\nu} \left( \frac{\kappa}{|\kappa|} \right)^n. \quad (6)$$

$d_i(\kappa)$  denotes the propagator of the exchanged quark ( $i = F, \overline{F}$ ) or gluon ( $i = G$ ).

The Regge singularities (branch points) are located at the point in  $\omega$  where two poles in  $\nu$  arising from the denominator pinch the  $\nu$  integration contour. Also the resummed anomalous dimensions  $\gamma(\omega)$  are obtained from this denominator. It is given by a zero  $\nu(\omega)$  such that  $\gamma(\omega) = \delta + i\nu[\omega]$ ,  $\delta = 1/2$  for GG or  $\delta = 0$  for  $F\overline{F}$  has a perturbative expansion, i.e. vanishes for  $g^2 \rightarrow 0$  [1].

The analogy to the BFKL case extends also to the remarkable properties of conformal symmetry, factorizability and integrability.

### 3 The double logarithmic approximation

Double logarithmic contributions arise in different situations and kinematical regions. The small  $x$  asymptotics of the GLAP evolution matches the large  $\kappa^2$  asymptotics of the BFKL evolution. Both are dominated by double logarithms from strong ordering in longitudinal and transverse momenta. The Regge asymptotics in the channels of equal chirality quark - antiquark exchange ( $F\overline{F}$ ) and odd parity two-gluon exchange ( $GG^\perp$ ) is dominated by double logs, whereby the transverse momentum integral is logarithmic in two regions (compare  $K_{F\overline{F}}$  in (3)). The GLAP evolution in these channels at small  $x$

accounts for only the logarithms from strongly ordered transverse momenta and therefore does not match the perturbative Regge behaviour [18].

Summing all double logs in these channels leads to involved linear integral equations in momentum representation. The method of separation of the softest particle leads directly to much simpler equations in terms of partial waves with a quadratic non-linearity [2]. One separates a loop in the configuration of transverse momenta in all loops where the one of the separated loop is the smallest. If the loop is built by a bremsstrahlung gluon Gribov's theorem allows to express the low  $\kappa$  contribution of this loop by contributions with the gluon radiated and absorbed by the external legs of the amplitude without this soft loop  $M(s, \kappa)$ . The second argument indicates that the transverse momenta in all loops are larger than  $\kappa$ . The bremsstrahlung contribution has the form

$$\int_{\mu^2}^s \frac{d\kappa^2}{\kappa^2} \left( \hat{m}_s \ln \frac{-s}{|\kappa^2|} + \hat{m}_u \ln \frac{s}{|\kappa^2|} \right) M(s, \kappa). \quad (7)$$

The gluon loop leads in particular to a change of the colour state in the  $t$ -channel. This is described by the matrices  $\hat{m}_s$  and  $\hat{m}_u$  depending to which external legs the gluon is attached. We change to partial waves

$$M^\pm(s, \kappa) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{|\kappa^2|} \right)^\omega \zeta^\pm(\omega) f^\pm(\omega). \quad (8)$$

The signature factor  $\zeta^\pm(\omega)$  behaves for small  $\omega$  as  $\zeta^+(\omega) \rightarrow 1, \zeta^-(\omega) \sim \omega$ . The positive signature contribution of the bremsstrahlung gluons to the partial wave is given by

$$(\hat{m}_s + \hat{m}_u) \frac{1}{\omega} \frac{d}{d\omega} f^{(+)}(\omega) \quad (9)$$

and the negative signature contribution by

$$\begin{aligned} & (\hat{m}_s + \hat{m}_u) \frac{1}{\omega^2} \frac{d}{d\omega} (\omega f^{(-)}(\omega)) \\ & - (\hat{m}_s - \hat{m}_u) \frac{1}{\omega^2} f^{(+)}(\omega). \end{aligned} \quad (10)$$

$\hat{m}_s + \hat{m}_u$  is diagonal and the element corresponding to the colour singlet channel vanishes. The positive signature part contributes as a inhomogeneous term to the negative signature channels, which follows from the small  $\omega$  behaviour of the signature factor.

The separated soft loop can also be built by a two-particle ( $F\bar{F}$  or  $GG^\perp$ ) intermediate state in the  $t$ -channel. This contribution to the amplitude is expressed by a loop integral involving two amplitudes of lower order. In terms of partial waves we obtain

$$\frac{1}{\omega} f^2(\omega). \quad (11)$$

This term conserves signature and all other  $t$ -channel quantum numbers.

It is easy to understand without explicit calculation that the exchange  $GG$  of two leading (BFKL type) gluons  $\mathcal{A}_+$  does not give rise to a double log contributions with these gluons carrying the smallest  $\kappa$ . By gauge invariance the amplitudes with external

$\mathcal{A}_+$  vanish proportional to the small transverse momentum of this gluon. Therefore there is no logarithm from the small transverse momentum integration range.

Now the Regge amplitude with the double logarithms of the types (7) and (11) summed up is the solution of the equation in Fig. 1. Starting from the Born term one iterates by adding bremsstrahlung loops and by inserting soft  $t$ -channel intermediate states. The Born term has the form  $\frac{g^2}{\omega}a_i$ . The coefficient  $a_i$  depends on the colour state in the  $t$ -channel. For the colour singlet state we have  $a_0 = C_2$  for  $F\bar{F}$  and  $a_0 = 4N$  for  $GG^\perp$  exchanges. In the parity odd flavour singlet case one has to consider the coupled channel of both exchanges with Born terms describing the transitions in the  $t$ -channel  $F\bar{F} \rightarrow F\bar{F}$ ,  $F\bar{F} \rightarrow GG^\perp$ ,  $GG^\perp \rightarrow F\bar{F}$ ,  $GG^\perp \rightarrow GG^\perp$ . The coefficients  $a_0$  coincide with the residues at  $j = 0$  of the corresponding one-loop anomalous dimensions.

Due to the mentioned structure of  $\hat{m}_s + \hat{m}_u$  the derivative terms (9) or (10) are absent in the colour singlet equation and we have just quadratic equations. We write the solutions for the  $F\bar{F}$  case.

$$\begin{aligned} f_0^{(+)}(\omega) &= 4\pi\omega \left( 1 - \sqrt{1 - \frac{g^2 C_2}{2\pi^2 \omega^2}} \right), \\ f_0^{(-)}(\omega) &= 4\pi\omega \\ &\left( 1 - \sqrt{1 - \frac{g^2 C_2}{2\pi^2 \omega^2} \left( 1 - \frac{1}{2\pi^2 \omega} f_0^{(+)}(\omega) \right)} \right). \end{aligned} \quad (12)$$

The solution for the colour octet channel  $f_V^{(+)}(\omega)$  enters the solution for the negative signature colour singlet partial wave. The corresponding equation is of Riccati type and the solution is expressed in terms of the parabolic cylinder functions.

The equation for the virtual Compton amplitude reduces to a linear equation with the (appropriate projection of the) Regge amplitude as the Born term. Therefore the large  $Q^2$  behaviour is determined by  $f_0^\pm(\omega)$  (12), which turn out to be (up to a factor  $(8\pi^2)^{-1}$ ) the asymptotic anomalous dimensions at  $j = \omega \rightarrow 0$ .

Notice that the Born coefficient  $a_0$  in the  $GG^\perp$  channel is large compared to the  $F\bar{F}$  channel. This implies for the position of the Regge singularities

$$\begin{aligned} \omega_{0GG^\perp}^{(+)} &= \left( \frac{4N}{C_2} \right)^{1/2} \omega_{0F\bar{F}}^{(+)}, \\ \omega_{0F\bar{F}}^{(+)} &= \left( \frac{g^2 C_2}{2\pi^2} \right)^{1/2}. \end{aligned} \quad (13)$$

The large  $\omega_{0GG^\perp}$  is the origin of the large effect of the double logs on the small  $x$  behaviour in the flavour singlet  $g_1(x, Q^2)$  obtained in [11].

This large effect demands to study the next-to leading log contributions. On the other hand it provides a qualitative understanding of the large gluon contribution to  $g_1(x, Q^2)$  which is observed in the EMC effect.

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